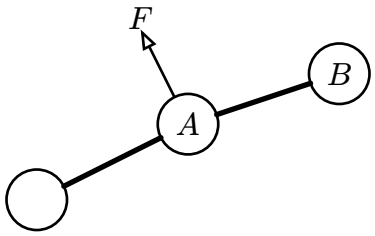
The Frequency of the Sling

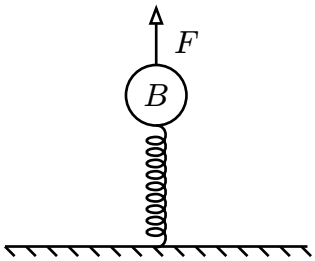
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This is a sling.



By experiment, I obtained that when the frequency of A and B differs from each other too much, B will soon stop moving.

Solution. It is clearly a stupid idea to directly solve the ODEs of this system, since there is no analytic solution to them. Hence, let's project the system onto the y-axis. Using A as the reference frame, we obtain that the motion of B in the vertical direction is approximately forced oscillation. Hence, we represent B's motion as the form below,



where $F = F_0 \cos \Omega t$, Ω is the angular frequency associated with the frequency of F, and $a_0 = \frac{F_0}{m}$ is the acceleration amplitude. It follows that

$$m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -ky - mg - F_0 \cos \Omega t.$$

The solution of this ODE is $y = \left(\frac{g}{\omega^2} - \frac{a_0}{\Omega^2 - \omega^2}\right)\cos \omega t - \frac{g}{\omega^2} + \left(\frac{a_0}{\Omega^2 - \omega^2}\right)\cos \Omega t$, where $\sqrt{\frac{k}{m}} = \omega$. Thus, we obtain

$$y = \left(\frac{g}{\omega^2} - \frac{a_0}{\Omega^2 - \omega^2}\right)\cos \omega t - \frac{g}{\omega^2} + \left(\frac{a_0}{\Omega^2 - \omega^2}\right)\cos \Omega t.$$

When $|\omega - \Omega| \gg 0$, $\frac{a_0}{\Omega^2} \gg \frac{g}{\omega^2}$. Hence,

$$y \approx \left(\frac{a_0}{\Omega^2}\right)(\cos \Omega t - \cos \omega t) - \frac{g}{\omega^2} = -2\left(\frac{a_0}{\Omega^2}\right)\sin\left(\frac{\omega + \Omega}{2}t\right)\sin\left(\frac{\Omega - \omega}{2}t\right) - \frac{g}{\omega^2}.$$

ω is a constant and $\omega \ll \Omega$ usually holds. Therefore, we obtain

$$y \approx -2\left(\frac{a_0}{\Omega^2}\right)\sin^2\left(\frac{\Omega}{2}t\right) - \frac{g}{\omega^2} = \left(\frac{a_0}{\Omega^2}\right)\cos(\Omega t) - \frac{a_0}{\Omega^2} - \frac{g}{\omega^2}.$$

Switch to the reference frame of the ground, we obtain

$$y' = y + \left(-\left(\frac{a_0}{\Omega^2}\right)\cos(\Omega t)\right) = -\frac{a_0}{\Omega^2} - \frac{g}{\omega^2},$$

this gives no oscillation, which is exactly what experiments give us.

Note 1. When $\omega = \Omega$, it follows that

$$y = \frac{g}{\omega^2}(\cos(\omega t) - 1),$$

Still, by switching to the reference frame of the ground, we obtain

$$y' = y + \left(-\left(\frac{a_0}{\Omega^2}\right)\cos(\Omega t)\right) = \frac{g}{\omega^2}(\cos(\omega t) - 1) - \left(\frac{a_0}{\Omega^2}\right)\cos(\Omega t).$$

Since $\omega = \Omega$, $a_0 \to 0$, it follows that

$$y' = \frac{g}{\omega^2}(\cos(\omega t) - 1),$$

which is a common oscillation.